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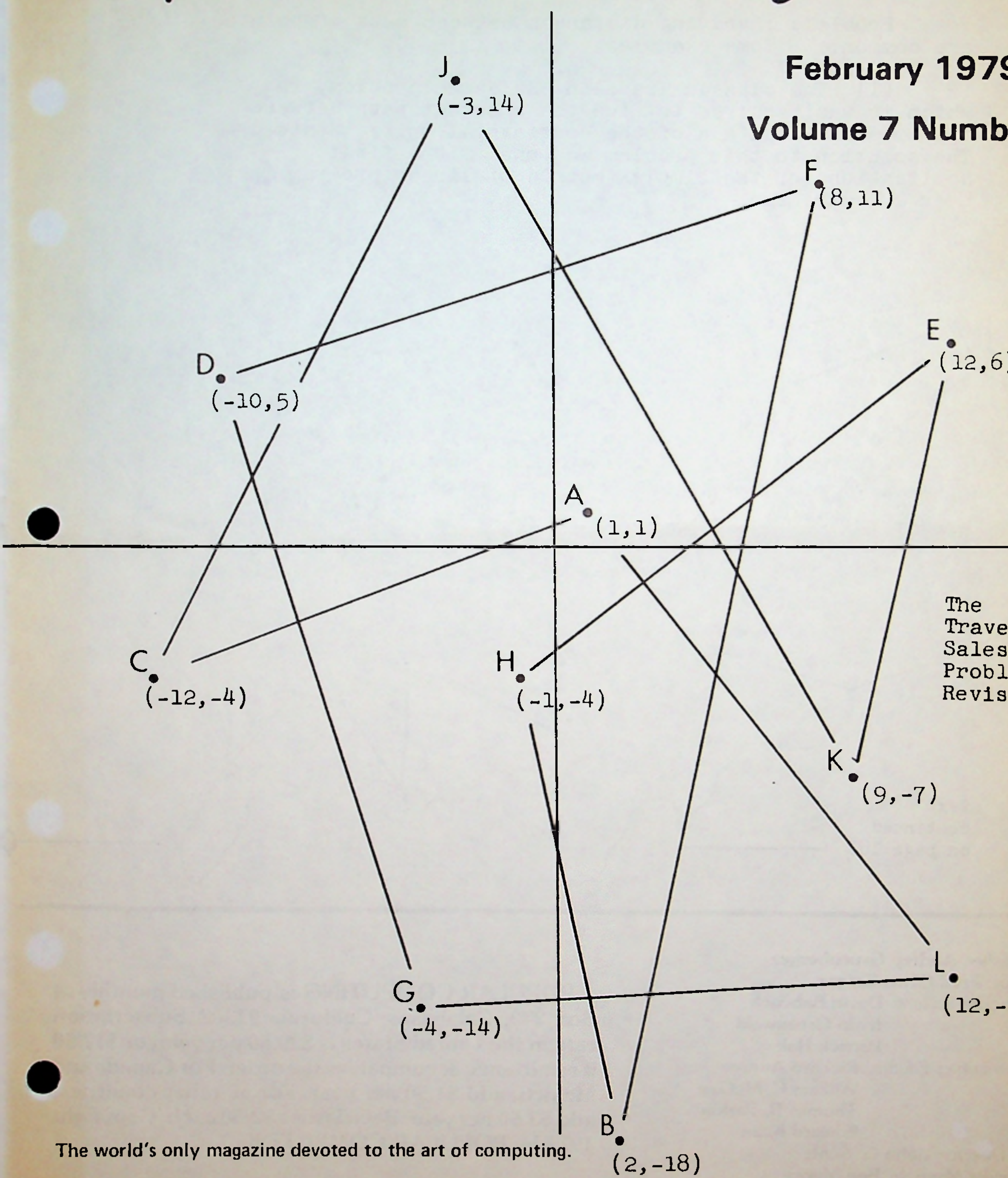


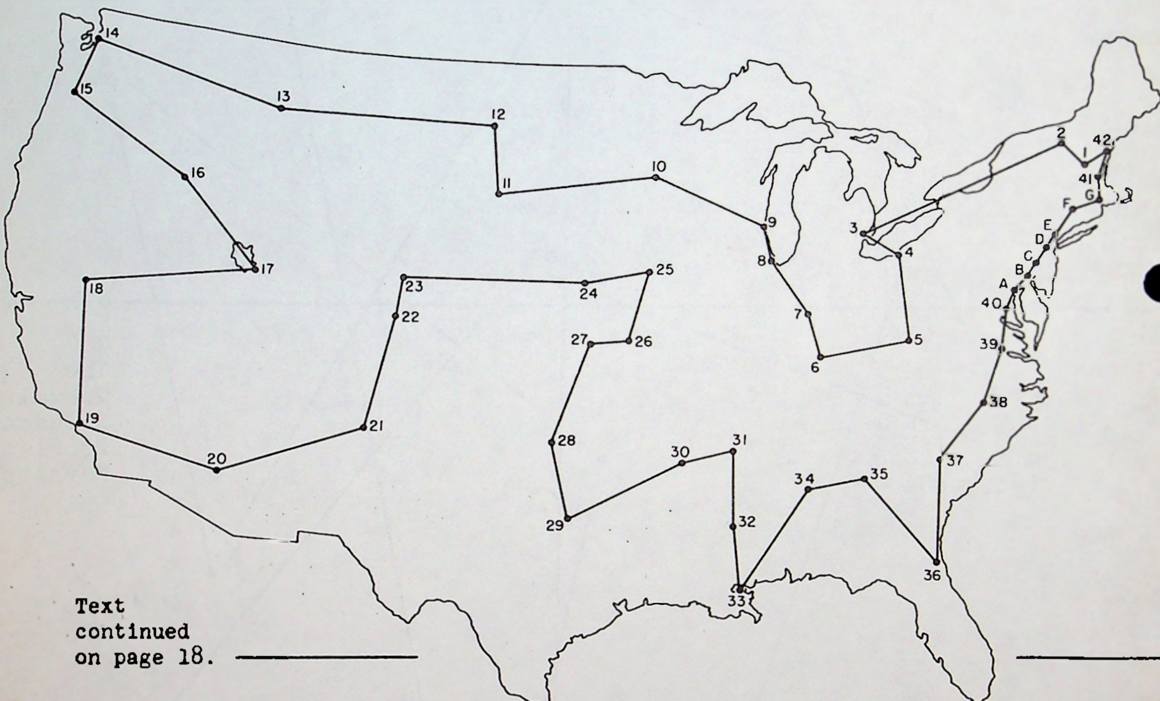
The Traveling Salesman Problem Revisited

The world's only magazine devoted to the art of computing.

The Traveling Salesman Problem Revisited

Problems involving distances between sets of points are common. Some examples:

(1) The classic Traveling Salesman problem, in which it was required to find the shortest path between the 48 state capitals of the continental United States. The solution to this problem was one of the first applications of the Simplex method of linear programming, over 20 years ago.



Text continued on page 18.

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Predictions

In our annual predictions in the April 1978 issue we said "A new largest prime number is found (December 1978)."

The record was actually broken October 30, 1978 by two 18-year old students, Laura Nickel and Curt Noll, at California State University, Hayward.

Using the Lucas-Lehmer test (described in detail in our issue No. 19), the new prime,

$$M = 2^{21701} - 1,$$

was found after

319 hours of CPU time (according to Nickel & Noll when they reported their finding on October 31 on the state university time-sharing network)

350 hours of CPU time (according to the report in Scientific American, in their January, 1979 issue)

440 hours of CPU time (according to the UPI story on November 16).

In any event, the 6533-digit number is shown intact on the next two pages.

On page 6 there is given the list of record breakers for the Mersenne primes; that is, those of the form

$$M = 2^p - 1, \quad p \text{ a prime.}$$

The exponent, p, is given in the second column. The index of that prime (that is, its position number in a table of primes, counting one as the first prime) is then given. The difference between successive indeces, in the fourth column, shows the number of attempts that had to be made as each new "largest prime" was revealed. (Nickel and Noll actually had to try only 74 values of p, rather than 181, since the previous record holder, Bryant Tuckerman, had tested all values of p up to 21000.)

The 25th Mersenne prime, $2^{21701} - 1$.

The number of 6533 digits fills these two pages.

448679166119043334794951410361591778727209023729388613010364
804475127856091580536371620183959201831086891496139730355336
211345516747152878800071343453471946810257320569398254237235
217504529801272150842995272668757068920072627984688251856815
321429857206372902993137263444632574164493445098351024588167
890163949458936967051685024361802322595516726032953891863644
370456813506975908621980471189012253526096503315606246416805
293609502763225195411993787816118879503680670654670945706027
039339445087959180179736113267982743384039648254487452704343
932588659353118262702812913017675373620735604717067960180869
861881923054773082631430336893094024011160231842187398617931
733816742936240390801464465331678931424034416262439863548298
681529638568560976139813700085452909021788527601812675169166
661891966528892724054575480157275832239363896906239482686297
384065953425368817706211940988842990422225102051989074276512
695908664890208493315931835447303784386736199054410756196653
620940877004175834722042393357617712606150632396871360582180
148553761288096481090253284446920609811491718531226232144903
151992703476798402506504466243611586327126073718204410761290
405760761564139819664433841749359455575892695505513844442496
417385893264310623502442597498884954275611131381253735467495
710892129770555341053063176680365254501156199947047030809750
018310390963281701931486716866592203695883622915919809638402
511820848390456430525809458761986972542352947185837167881659
762285573752842609723773462446853236203269278121288691228046
449177079431863844760577118105582286531205728018099382507023
117893201429792131750687465985253431776947579925010154034590
557560961501317963693425717129169735471664746988565921326127
172442496858615424105976103547295501253805844971090564639919
264483572232090088277647132202284397757572307555166586140282
929609475227094567511997522684243830841903167699758289694081
615129297311661887746319739426130035053280305284359298619430
288146472243934096043321299157094797164800017511529010477300
220992264963936226333454050093611469551563389634481137856461
309799339035379297516197686695505630024237061612366915362995
773054421160559104626824057283049330226956348760335479181171
767044316513727297536114852079572830976077659331692529138612
877268338300007559071684963736432018866272260591868249870461
589502801214803101062208217608651205629204397566580811536177
263527091318013871586725598473961011785486552173088901587886
093243264279191781217083260426698576752827096096779295327807
380600017055574112385649459728735921101312391161744629474242
794027578530736502060580720891390808133815521202378891769590
327255321879818063901889766119669233837800805895711803041279
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973081518846972691730480391423511164600093411354656825597954
514070657571647126828926843403999279968415403789437874392343
825455622539733000039772914979455803684686423137847235539529
117541151083852900548655637845231613259111843384039750925611
825414884470597441712363488652928979195070962474324857427580
967098521325532436870880026366373568836472092813844664683486
614735394776761147139523905742838711386436089276896462702654
362270723613615569693260269725551377958061315120783963923811
171630214883197191229539672418496500356865168976365328298294

1355943827107890964801254681815583067064973102592456105207 95
1836551169131591035954957809854202185085886019569497663035 13
66074278129451101792937644085504395275589852388064138602 4393
46909569031381270272833205694814223411334423565977453887 5423
95353485301546284835327702006873689327611630437966491140 0779
51840154972590243416512380587609900111470825767310676939 4419
42826834427612997756892340669203408743659224466633723212 5966
50417249730825215118303577011272768581293371366148392778 5323
35573122542905472394542069919985611732624482347646980930 8055
86007081976261003097963198812882873782621377463926668786 4982
04922447802136607398552526766227205124310008056632919363 5531
94966717056709446690809053625020657218419836547624618595 3753
52039018705449062959037038925284556675116361168237279922 2079
20794703117075921951295707710873733317931433110923531405 7194
16365240907518118794096775462324423434250371711420947553 5715
55908465792580645089870615111865933553809003552706994499 7487
06917899889082891225016257753453229705906397483096836459 6776
40750701051127649136815511445021677568414826304009394300 8537
62240103073543616073297109679097852591173721376034038403 9037
97326084264850909622924265141840684883798695123898530601 7136
27917934581206083528373369371952855787986368194089909153 0377
60311683730584329384267492189510884229753903906808988993 5232
59437606901944039838823315225488711871355530334671879703 2874
93612171018890720047791219983006575690550227543246901734 7490
51876793980240930546447655515405860615566182339568500526 0806
85358056916078877218441535928716267560418132595102517522 9289
81353833074067246933111570873295331815501725771241383698 6051
05802286768924861382741652271179538064625778814288737402 1012
20075694269814240934893379113612806306410045157701609671 5175
43787442511524203121293023260598375610136932287924468473 5696
26893292863959506611559181514200293625478634965230187600 9932
04860736747921056014359652196606989325320755082356901630 6692
40688685505174445515739151017185696401582807173431580481 6271
22423292641208991071222953038343766541925054971737959169 8692
92375645057151727543470106206531135735154136115144300015 8730
80448570355596137321380062748881332031472142823024949510 0224
09370177712994235289702790490255826941441774321618772576 4552
98250619988354955614763200793811728997204477597638052792 5108
78225745663494703701392574625552528908681553926116252158 2439
53964293657171414581341823547346468074289772645946104841 1808
01333621512150598625594195401601933368973292090945435953 3018
75202828322848138663871822530516790587144451339179411203 3281
34884327892252621182901800244630979609876813374281202935 9981
28401633959108387881070976339195584757402099414224321539 5434
76019604874975720230127703172732861447718691961098975942 3327
52633167063512960112700154075209382155551695503282689329 4297
89346503176351007528609841743736084042408094015850910964 6184
82805649640269394165006760210370817982850495571836159005 7099
17919149733735278914364020463842602726375487527643414975 6920
20200079346255666615166515258291939313433912222614621242 0141
53365037286833662921186290423547789663783785467893012638 0410
82143785487398866487992341179948504338667781255945413472 4652
46231194881401316071628427281713042247869185631200192333 6989
66933544361629391311041730956501694662754558875644345191 2692
796006935518092719564502642940928574108283535118827 51

	p	Index	Number of primes tried	
6	17	8		
7	19	9	1	
8	31	12	3	
9	61	19	7	
10	89	25	6	
11	107	29	4	
12	127	32	3 *	
13	521	99	67	
14	607	112	13 *	
15	1279	208	96	
16	2203	329	121	
17	2281	340	11 *	
18	3217	456	116	
19	4253	584	128	
20	4423	603	19 *	
21	9689	1197	594	
22	9941	1227	30 *	
23	11213	1358	131	
24	19937	2255	897	8 years here
25	21701	2436	181	7 1/2 years here

A listing of the 6th through 25th Mersenne primes, together with a crude measure of the computational work involved in establishing each new record.

Occasionally, this number is small (the entries marked with *), and a new record breaker is found with only modest effort. On the other hand, when that number is large (as was the 897 leading to the record held by Tuckerman for 7 1/2 years), then a great deal of discouraging work must be done before success scored. It is clear from the table why p = 127 held the record for many years. The M value for p = 127 has 39 digits, but for p = 521 it has 157 digits. To find the M for p = 521 represented over 1200 times as much computation as was involved in finding the M for p = 127, and the later record had to await the use of computers.

Note that there is no assurance that there will <u>be</u> a next Mersenne prime. But if there is, will the next entry in the "number tried" column be small or large?

In our April issue, we had also this prediction:

> "A <u>new</u> 'new largest prime number' (this one of 9000 digits) is found (March 1980)."

We will stick largely by that prediction, but amplify it:

(a) Approximately 128 values of p will have to be tested.

(b) The new p will be around 22973.

(c) The number of digits in the new M will thus be around 6900.

(d) The computational work for the Lucas-Lehmer test is proportional to the cube of p. Thus, for the next record breaker, the total amount of computing power needed will be approximately twice that needed for the current record.

(e) The next record will be established on a personal (home) computer of the size and power of an Apple II having 16K bytes of storage.

(f) Several owners of such machines will unite to parcel out the work, and the total project time will cover about 4 months.

(g) ...and <u>that</u> record will be broken within two years, again on home machines.

SignDEAFined

by David Babcock

In POPULAR COMPUTING's issue number 23 there was a description of a project carried out at California State University, Northridge to aid the deaf. Ostensibly, it was to aid deaf students of computing, but actually the purpose of the project was to explore and demonstrate a proper technique for the dissemination of the hand signs that are widely used in communicating with the deaf. That project developed 120 signs for the most common terms used in computing, and displayed them in color motion pictures for ease of learning.

The problem of communication with the deaf is huge, largely unexplored, and, until recently, much neglected. Deafness, as a handicap to earning a living, is probably worse than blindness. Moreover, as Jerome Shein and Marcus Delk point out in The Deaf Population of the United States, "impairment of hearing is the single most prevalent chronic physical disability in the United States."

Defining prevocationally deaf as those persons who could not hear and understand speech and who had lost (or never had) that ability prior to 19 years of age, the National Census of the Deaf Population has calculated the number of prevocationally deaf as 203 per 100,000 population (about half a million, then, for our current population). It is estimated that those with hearing impairment average 72% of the income of similar groups who have normal hearing. Among non-whites with impaired hearing, the figure is 62% of normal. "For most of those who are afflicted, deafness is expensive."

A large factor in the problem is the simple inability to communicate between the deaf and the hearing, and among the deaf. Training in lip reading or hand signing is slow and expensive, and the end result is not quite satisfactory, to the end that those with impaired hearing have difficulty in learning any subject. The project at Northridge was designed to solve one small part of the problem; namely, how to transmit hand signs among the people concerned with them, with maximum efficiency. Alternative methods of transmission (drawing multiple cartoons, person-to-person training, or video tape) help, but do not do a good job, or are much too expensive.

In the film project, a given term, like "accumulator," was first finger-spelled, then given by its new sign, with the term shown in English at the bottom of the screen, then repeated from the point of view of the signer; that is, 180 degrees away from the viewpoint of the receiver. The films established the proper colors of clothes and background, and the speed of presentation at which the signs could be learned most readily. The four 6-minute films were made available in 1974, but were not very successful. The problems remain, both of standardizing the signs and of disseminating them. While color movies might do the latter job, there is a better way.

The computer, as a device to execute complex protocols and produce elaborate pictures on a display, is the natural tool to use to disseminate signs for the deaf. Suppose that a sign was to be taught for a word like "friend." A program in the computer generates a display to present the sign on a CRT, in motion. The sign can be shown (as in the films) from the front and from the rear. But now we can go much further, since the basic information is stored and can be manipulated. With computer control, the sign could be viewed from any desired angle. It could be presented at any desired speed, or be frozen on the screen. Zooming, panning, and close-ups are all possible techniques.

As a training tool, to teach a vocabulary of signs, the machine can run through a collection of signs in any desired order (including random order): alphabetically by subject; alphabetically across subjects; in order of frequency of use in any subject; and so on. The computer can play "What Sign Am I?" for efficient drill and self-test modes of learning. Any given sign can be looked up and displayed. Information can be transmitted with the manual alphabet (finger spelling). The computer-controlled display acts as a personal tutor, one-on-one and highly interactive. In sum, the computer can offer flexibility and versatility far beyond what film can do.

There are some interesting side effects, too. A new sign can be distributed quickly to all installations via an inexpensive cassette, thus offering speed of dissemination as well as near-zero cost (after the initial programming cost is amortized). The economies of scale would take effect quickly in this mode of communication. Thus, a new sign could become a standard sign in short order.

The capability needed for all that has been described matches that of the Apple II computer with a disk and 48K bytes of storage, for example. In addition to all the other advantages over film, there is the magic of the computer still at work; that is, the proposed system appears to the user in terms of "The computer says that the sign is..." And on top of all that, during periods in which the computer is not being used for sign language, its power as a computational tool is still available for all other purposes.

ART OF COMPUTING 16

WHICH PROGRAM IS THE BEST?

If we have two or more programs that produce the same outputs from the same inputs (that is, that appear to perform the same task), which of them could be rated the best in some sense? Some of the ways in which programs might be compared and rated are:

1. The amount of CPU time they consume to do their work.

2. The amount of CPU storage space they consume for instructions and data.

3. The man-hours consumed in writing, debugging, and testing the program.

4. The amount of CPU time spent in debugging and testing the program.

5. The sum of items (3) and (4).

6. The elapsed time between the assignment of the problem to the programmer and the delivery of a tested and documented program.

7. The ease with which someone else can modify or correct the program.

It is rarely true that one of these criteria will consistently outweigh the others, and the differences between them should surely constitute one of the basic topics to be considered in an introductory course in computing.

On the assumption that computing installations are well-run and stable, then most of the time in today's EDP world criterion (7), MAINTAINABILITY, dominates the list. Please note all the disclaimers in that statement. The percentage of well-run and stable computing installations is not awfully large.

For some mysterious reason, these notions (which seem fairly obvious to experienced people) require considerable explaining to most beginners. In particular, the concept of elapsed time (6) and its importance, deserves much attention. It is all too frequently the case in industry that by the time a problem situation becomes apparent, it is already an urgent problem, if not a panic situation. If the situation is such that the company will go out of business if this problem is not solved immediately, then considerations of excessive CPU time, or programmer overtime, or maintenance of the program, fade into insignificance.

Let's see if we can make that concept more vivid with a real-life example.

Consider a plutonium reactor. Such an atomic pile is a huge block of pure graphite, through which pass many aluminum tubes. The tubes are loaded with canned cylinders of uranium, surrounded by pure, cold water. When sufficient uranium has been inserted into the graphite block, the reactor goes critical and proceeds to cook the uranium, which turns part of it into plutonium. The cold water cools the furnace and also serves to moderate the atomic reaction.

One day, after a number of such reactors had been operating for some years, one of the reactors sprang a leak. One of the aluminum tubes had ruptured, spraying water all over the inside of the graphite block, and immediately bringing the plutonium-producing process to a halt. It soon became evident that the aluminum tube had simply worn out from the flow of water through it.

This is not the planned and graceful way that a reactor is usually halted, and the subsequent drying-out process is long and costly. The cost of a plutonium reactor is in the hundreds of millions of dollars bracket, and when one is down, it is a worthless device.

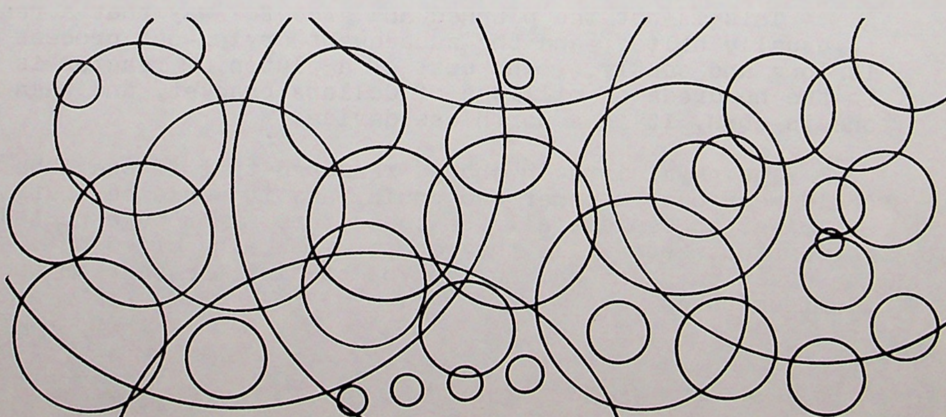
The physicists in charge reasoned that if one tube could wear out, another one could, and it would be most efficient to replace all the tubes that might be expected to wear out soon while the reactor was being dried out. The question then was: how do you predict the wear on those aluminum tubes?

As it turned out, someone had derived a formula for tube thickness, given the original thickness, and the temperature and pressure of the water that had flowed through the tube. It was also true that the water pressure and temperatures (two of the latter--the incoming cold temperature and the discharge high temperature) had been recorded and keypunched for every hour that the reactors had been operated (this came to some millions of punched cards).

All that had to be done was a job of data reduction; namely, segregate all the data for each individual tube, apply it to the wear formula, and print out lists of the tubes in wall thickness order, thinnest first. There was one complicating factor: the format of the punched cards had clearly been changed several times over the years, and there was no record of any card layout.

The team assigned to this data processing problem had clear priority over every other task in the plant. Every minute that it took to produce a debugged and tested program was one minute less that the reactor would be up, not to mention the urgent feeling that other reactors were probably ready to go down for the same reason.

Here is one clear instance in which any technique that worked was the right technique. The program did not have to be elegant, or even neat. It could be rewritten later for neatness and maintainability, but for the moment it needed only to get running correctly. It was a perfect example of the situation where elapsed time is the overriding criterion.



Penny Flipping Again

We first discussed the Penny Flipping problems back in issue 23 (February 1975). Recently, more work on the original problem has turned up some interesting results. We restate the original problem (due to Iain Bride and John Gilder) as it appeared in the book SIMULA BEGIN:

> A pile of C pennies is arranged so that each penny is heads up. We define an operation FLIP(Q) on this pile, which removes the top sub-pile of Q pennies, turns the sub-pile upside down and replaces them on top of C - Q pennies remaining. The consecutive operations:
>
> FLIP(1),FLIP(2),...,FLIP(C),FLIP(1),FLIP(2),...
>
> are repeated tuntil the stack returns to all heads. The value of N for a given C is the number of flips required to return the stack to heads. For C = 6, for example, N = 35.

Associate editor David Babcock extended the list of results to case C = 240, as shown in the accompanying table. Each result so far can be put into one of four categories. For case C, the number of flips is one of:

(1) kC (2) C^2 (3) $C^2 - 1$ (4) kC - 1

and the appropriate case is indicated in the table. There seems to be no predictable pattern, and certainly no proof that there might not be a result different from the four listed categories. So the original problem (designed to show off the power of the language SIMULA) leads to some interesting new problems that could sop up otherwise idle time on a lot of personal computers:

(1) Does the pattern of four types continue indefinitely?

(2) If it does, is there any way to predict the result for case C as a function of C? In other words, do the x's of our table form a pattern?

(3) Beginning at C = 241, the cases are averaging 28000 flips each, which begins to get costly in terms of CPU time. Flowcharts are given for one scheme of solution of the problem--can their logic be improved?

PROBLEM 253

C		1	2	3	4
1	2	x			
2	3			x	
3	9		x		
4	11				x
5	24			x	
6	35			x	
7	28	x			
8	31				x
9	80			x	
10	60	x			
11	121		x		
12	119				x
13	116				x
14	195			x	
15	75	x			
16	79				x
17	204	x			
18	323			x	
19	228	x			
20	199				x
21	146				x
22	264	x			
23	529		x		
24	504	x			
25	200	x			
26	675			x	
27	540	x			
28	251				x
29	840			x	
30	899			x	
31	186	x			
32	191				x
33	1088			x	
34	748	x			
35	1225		x		
36	324	x			
37	740	x			
38	1140	x			
39	1521		x		
40	1079				x

C		1	2	3	4
41	1680			x	
42	336	x			
43	1204	x			
44	484	x			
45	540	x			
46	460	x			
47	1692	x			
48	1151				x
49	734				x
50	2499			x	
51	2601		x		
52	624	x			
53	2808			x	
54	971				x
55	1980	x			
56	783				x
57	2508	x			
58	696	x			
59	1415	x			
60	3299				x
61	1220	x			
62	3099				x
63	441	x			
64	447				x
65	4224			x	
66	1188	x			
67	2412	x			
68	2311				x
69	4760			x	
70	3220	x			
71	4260	x			
72	1007				x
73	3066	x			
74	5475			x	
75	1125	x			
76	1824	x			
77	1540	x			
78	2027				x
79	4108	x			
80	2640	x			

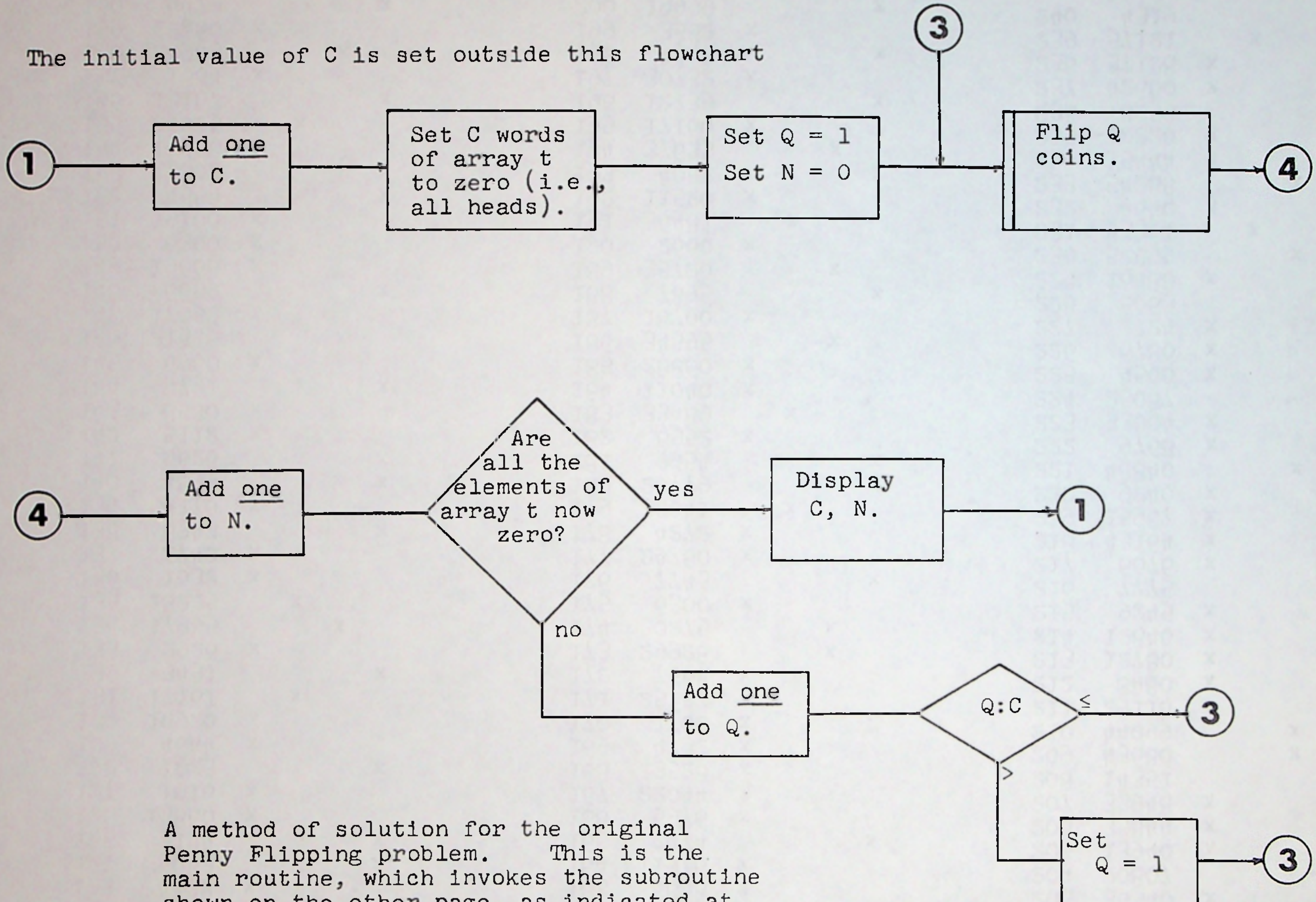
C		1	2	3	4
81	6560			x	
82	1640	x			
83	6889		x		
84	6551				x
85	764				x
86	7395			x	
87	5220	x			
88	2551				x
89	7920			x	
90	8099			x	
91	5460	x			
92	1655				x
93	3720	x			
94	1692	x			
95	9025		x		
96	4607				x
97	1164	x			
98	9603			x	
99	9801		x		
100	3299				x
101	8484	x			
102	1019				x
103	6798	x			
104	4679				x
105	11024			x	
106	7420	x			
107	2996	x			
108	1620	x			
109	1962	x			
110	2640	x			
111	4107	x			
112	6720	x			
113	12768			x	
114	4331				x
115	3450	x			
116	3364	x			
117	10764	x			
118	9204	x			
119	14161		x		
120	1439				x

C		1	2	3	4
121	9800				x
122	10248	x			
123	4428	x			
124	5083				x
125	3124				x
126	13860	x			
127	1016	x			
128	1023				x
129	4644	x			
130	10920	x			
131	17161		x		
132	3431				x
133	2926	x			
134	17955			x	
135	18225		x		
136	1632	x			
137	2740	x			
138	6347				x
139	4170	x			
140	4899				x
141	6626				x
142	5112	x			
143	8580	x			
144	9791				x
145	6960	x			
146	21315			x	
147	17052	x			
148	6659				x
149	19668	x			
150	6300	x			
151	15100	x			
152	4559				x
153	7802				x
154	15708	x			
155	24025		x		
156	12167				x
157	1884	x			
158	24963			x	
159	22260	x			
160	8479				x

C		1	2	3	4
161	11592	x			
162	4859				x
163	5868	x			
164	11316	x			
165	2474				x
166	5976	x			
167	22044	x			
168	3528	x			
169	4732	x			
170	1700	x			
171	25137	x			
172	7568	x			
173	29928			x	
174	30275			x	
175	6300	x			
176	7743				x
177	24780	x			
178	4272	x			
179	32041		x		
180	20779				x
181	9954				x
182	6552	x			
183	33489		x		
184	11040	x			
185	28860	x			
186	34595			x	
187	18700	x			
188	7895				x
189	35720			x	
190	2660	x			
191	36481		x		
192	11520	x			
193	4052				x
194	37635			x	
195	17160	x			
196	12739				x
197	30732	x			
198	4355				x
199	3582	x			
200	19999				x

C		1	2	3	4
201	12060	x			
202	21816	x			
203	36540	x			
204	20807				x
205	13940	x			
206	35844	x			
207	33948	x			
208	14351				x
209	43680			x	
210	44099			x	
211	29118	x			
212	8480	x			
213	12780	x			
214	12840	x			
215	9245	x			
216	7775				x
217	6076	x			
218	43164	x			
219	15987	x			
220	9240	x			
221	48840			x	
222	9768	x			
223	33004	x			
224	25087				x
225	4500	x			
226	6780	x			
227	2724	x			
228	8663				x
229	16488	x			
230	52899			x	
231	53361		x		
232	4640	x			
233	54288			x	
234	15444	x			
235	12220	x			
236	8259				x
237	42660	x			
238	37128	x			
239	57121		x		
240	4319				x

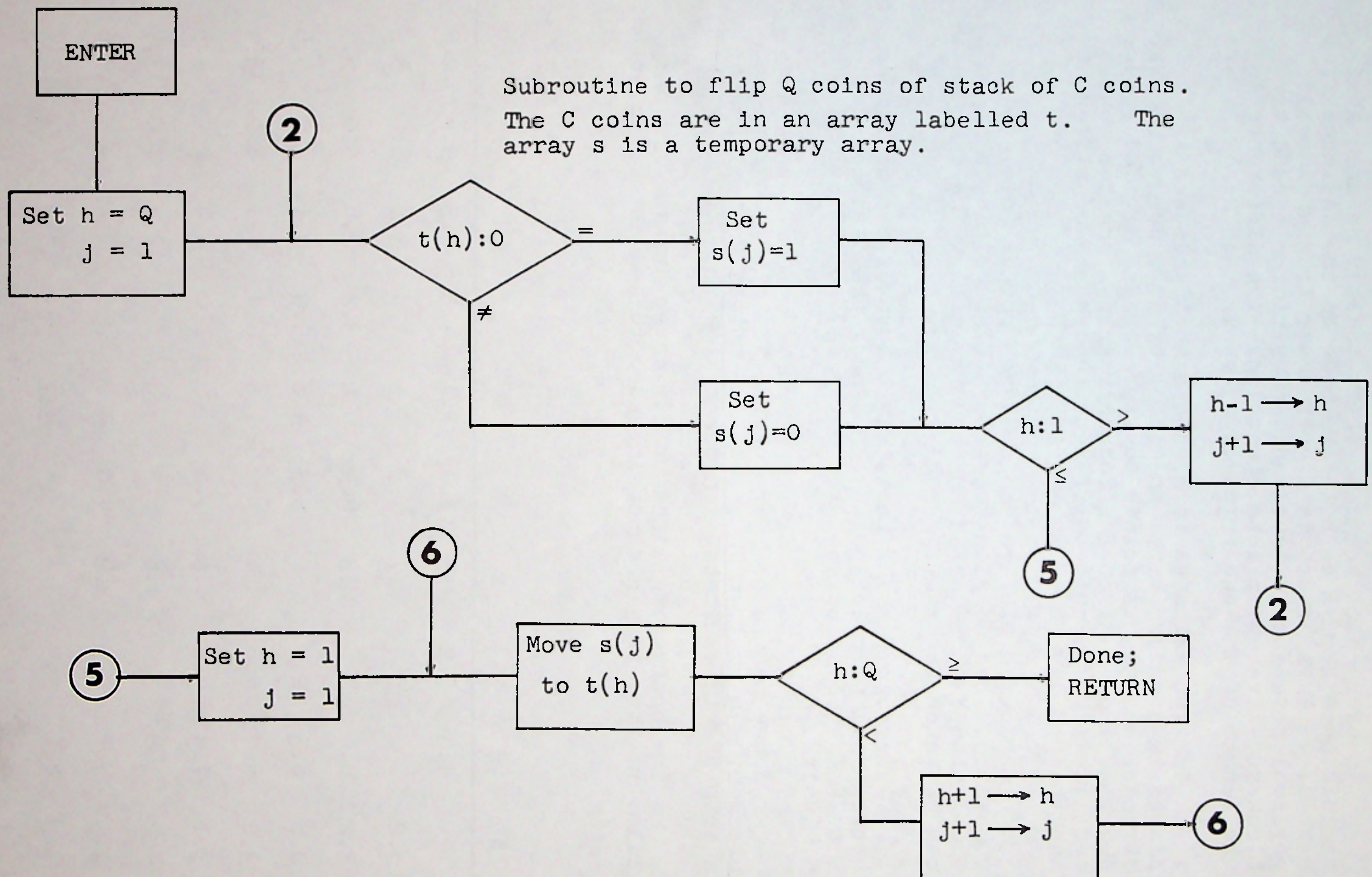
The initial value of C is set outside this flowchart



A method of solution for the original Penny Flipping problem. This is the main routine, which invokes the subroutine shown on the other page, as indicated at Reference 3.

Subroutine to flip Q coins of stack of C coins.

The C coins are in an array labelled t. The array s is a temporary array.



...Continued from page 2

(2) Our own Disgruntled Crew's Cruise (Problem 68, Issue No. 20, November 1974), in which it was required to find the longest possible trip between 12 points.

In every such problem, it is tacitly assumed that the distance from A to B is the same as the distance from B to A. This "fact" is almost never true when real physical movement is involved. Try riding around the block you live on in both directions--the difference in the length of the two trips might be as much as 6%.

The two trips (A to B, and B to A) between two places in a modern city can differ by 10%, due to one-way streets, on and off points to expressways, divided roads, and the like. Even for long air trips, the two routes are likely to be quite different, and hence of different length.

Our cover picture shows eleven points (cities, if you wish), located with reference to an origin as shown by the X- and Y-coordinates in parentheses. The coordinates can be taken as exact.

The accompanying table gives both of the distances between all pairs of points. Thus, the distance from D to K is 5.6 units, but the distance from K to D is 5.9 units.

So we have a new form of the Traveling Salesman problem: In what order should the points be visited for a trip of minimum total length?

The connecting lines on our cover drawing are only illustrative; they are not intended to indicate a solution.

	A	B	C	D	E	F	G	H	J	K	L
A		4.3	3.9	2.9	3.0	3.1	4.3	1.3	3.4	2.9	4.6
B	5.2		5.0	6.7	6.3	7.5	1.8	3.5	8.0	3.3	2.8
C	3.2	5.0		2.3	6.5	6.2	3.2	2.8	5.0	5.3	6.5
D	2.8	6.3	2.4		5.5	4.8	5.0	3.1	2.9	5.6	7.1
E	3.2	6.7	6.6	5.4		1.6	6.4	4.1	4.2	3.4	4.8
F	3.1	7.4	6.1	4.6	1.6		7.0	4.4	2.9	4.5	6.1
G	4.2	1.8	3.3	4.8	6.3	6.8		2.7	7.0	3.7	4.0
H	1.5	3.6	2.7	3.3	4.0	4.7	2.5		4.5	2.6	3.9
J	3.5	8.2	4.8	3.1	4.4	2.7	6.7	4.4		6.0	7.8
K	2.7	3.1	5.0	5.9	3.7	4.7	3.8	2.4	6.3		1.7
L	4.4	2.7	6.6	7.0	4.7	6.0	4.1	4.1	7.5	1.7	

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